An oscillating, homogeneous and isotropic Universe from Scalar-Tensor gravity

Christian Corda

August 26, 2008

Aerospace Tecnology Center, Av. Segle XXI s/n - Parc De La Marina, E - 08840 Viladecans - Barcelona, Spain and Associazione Galileo Galilei, Via Pier Cironi 16 - 59100 PRATO, Italy

E-mail address: christian.corda@ego-gw.it

Abstract

An oscillating, homogeneous and isotropic Universe which arises from Scalar-Tensor gravity is discussed in the linearized approach, showing that some observative evidences like the Hubble Law and the Cosmological Redshift are in agreement with the model. In this context Dark Energy appears like a pure curvature effect arising by the scalar field.

PACS numbers: 04.80.Nn, 04.30.Nk, 04.50.+h

1 Introduction

The accelerated expansion of the Universe, which is today observed, shows that cosmological dynamic is dominated by the so called Dark Energy which gives a large negative pressure. This is the standard picture, in which such new ingredient is considered as a source of the *rhs* of the field equations. It should be some form of un-clustered non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so called "concordance model" (ΛCDM) which gives, in agreement with the CMBR, LSS and SNeIa data, a good trapestry of the today observed Universe, but presents several shortcomings as the well known "coincidence" and "cosmological constant" problems [1].

An alternative approach is changing the *lhs* of the field equations, seeing if observed cosmic dynamics can be achieved extending General Relativity [2, 3, 4]. In this different context, it is not required to find out candidates for Dark Energy and Dark Matter, that, till now, have not been found, but only the "observed" ingredients, which are curvature and baryonic matter, have to be taken into account. Considering this point of view, one can think that gravity is not scale-invariant [5] and a room for alternative theories is present [6, 7, 8]. In principle, the most popular Dark Energy and Dark Matter models can be achieved considering $f(R)$ theories of gravity [5, 9], where R is the Ricci curvature scalar. $f(R)$ theories of gravity are also conformally coupled with Scalar-Tensor theories of gravity [3, 4, 5, 6, 7, 9]. In this picture even the sensitive detectors for gravitational waves, like bars and interferometers (i.e. those which are currently in operation and the ones which are in a phase of planning and proposal stages) [10, 11], could, in principle, be important to confirm or ruling out the physical consistency of General Relativity or of any other theory of gravitation. This is because, in the context of Extended Theories of Gravity, some differences between General Relativity and the others theories can be pointed out starting by the linearized theory of gravity [12, 13, 14, 15].

This paper is an integration of my previous research on oscillating Universes [7]. In [7] it as been shown that an oscillating, homogeneous and isotropic Universe which arises from the linearized R^2 theory of gravity is fine-tuned with some observative evidences like the cosmological redshift and the Hubble law. In this paper an oscillating, homogeneous and isotropic Universe which arises by the linearized Scalar-Tensor gravity, recently analysed from the point of view of gravitational waves in [12, 15], is discussed, showing that the same observative evidences, i.e. the Hubble Law and the Cosmological Redshift, are in agreement with the model in this case too. In this context Dark Energy appears like a pure curvature effect arising by the scalar field. The paper is organized in this way: in the second section the linearization of Scalar-Tensor gravity is performed, showing that a third mode of gravitational radiation arises from the action

$$S = \int d^4x \sqrt{-g}[f(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) + \mathcal{L}_m], \qquad (1)$$

which is the Scalar-Tensor modification with respect the well known canonical one of General Relativity (the Einstein - Hilbert action [16, 17]), i.e.

$$S = \int d^4x \sqrt{-g}R + \mathcal{L}_m. \qquad (2)$$

In the third section, with the assumption that this third mode becomes dominant at cosmological scales, following the linees of [7] an oscillating model of Universe will be shown.

In the fourth and fifth sections, the tuning with the Hubble Law and the Cosmologic Redshift will be analysed.

2　The linearized Scalar-Tensor gravity

Choosing

$$\varphi = f(\phi) \quad \omega(\varphi) = \frac{f(\phi)}{2' f(\phi)} \quad W(\varphi) = V(\phi(\varphi)) \tag{3}$$

eq. (1) reads

$$S = \int d^4x \sqrt{-g} [\varphi R - \frac{\omega(\varphi)}{\varphi} g^{\mu\nu} \varphi_{;\mu} \varphi_{;\nu} - W(\varphi) + \mathcal{L}_m], \tag{4}$$

which is a generalization of the Brans-Dicke theory [23].

By varying the action (4) with respect to $g_{\mu\nu}$ and the scalar field φ, the field equations are obtained (i.e. in this paper we work with $G = 1$, $c = 1$ and $\hbar = 1$) [6, 12]:

$$G_{\mu\nu} = -\frac{4\pi\tilde{G}}{\varphi} T_{\mu\nu}^{(m)} + \frac{\omega(\varphi)}{\varphi^2} (\varphi_{;\mu}\varphi_{;\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \varphi_{;\alpha}\varphi_{;\beta}) +$$

$$+ \frac{1}{\varphi}(\varphi_{;\mu\nu} - g_{\mu\nu}\Box\varphi) + \frac{1}{2\varphi} g_{\mu\nu} W(\varphi) \tag{5}$$

with associed a Klein - Gordon equation for the scalar field

$$\Box\varphi = \frac{1}{2\omega(\varphi)+3}(-4\pi\tilde{G}T^{(m)} + 2W(\varphi) + \varphi W'(\varphi) + \frac{d\omega(\varphi)}{d\varphi} g^{\mu\nu} \varphi_{;\mu}\varphi_{;\nu}. \tag{6}$$

In the above equations $T_{\mu\nu}^{(m)}$ is the ordinary stress-energy tensor of the matter and $\tilde{G}$ is a dimensional, strictly positive, constant [6, 12]. The Newton constant is replaced by the effective coupling

$$G_{eff} = -\frac{1}{2\varphi}, \tag{7}$$

which is, in general, different from G. General Relativity is recovered when the scalar field coupling is

$$\varphi = const = -\frac{1}{2}. \tag{8}$$

Because we want to study interactions at cosmological scales, the linearized theory in vacuum ($T_{\mu\nu}^{(m)} = 0$), which gives a better approximation than Newtonian theory, can be analyzed, with a little perturbation of the background, which is assumed given by a Minkowskian background plus $\varphi = \varphi_0$. Because we are limiting ourself to the linar approximation, ω can be considered constant and fixed by φ_0 [6, 12]:

$$\omega = \omega_0 = \omega(\varphi_0). \tag{9}$$

φ_0 is also assumed to be a minimum for W:

$$W \simeq \frac{1}{2}\alpha\delta\varphi^2 \Rightarrow W' \simeq \alpha\delta\varphi. \tag{10}$$

Thus, putting

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$\varphi = \varphi_0 + \delta\varphi, \tag{11}$$

to first order in $h_{\mu\nu}$ and $\delta\varphi$, calling $\widetilde{R}_{\mu\nu\rho\sigma}$, $\widetilde{R}_{\mu\nu}$ and $\widetilde{R}$ the linearized quantity which correspond to $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R, the linearized field equations are obtained [6, 12]:

$$\widetilde{R}_{\mu\nu} - \frac{\widetilde{R}}{2}\eta_{\mu\nu} = \partial_\mu\partial_\nu\xi - \eta_{\mu\nu}\Box\xi$$
$$\Box\xi = -E^2\xi, \tag{12}$$

where

$$\xi \equiv \frac{\delta\varphi}{\varphi_0}$$
$$E^2 \equiv \frac{\alpha\varphi_0}{2\omega_0 + 3} \tag{13}$$

have been defined. E represents the "curvature" energy wich arises from the scalar field.

$\widetilde{R}_{\mu\nu\rho\sigma}$ and eqs. (12) are invariants for gauge transformations

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} - \partial_{(\mu}\epsilon_{\nu)}$$
$$\delta\varphi \to \delta\varphi' = \delta\varphi; \tag{14}$$

then one can define

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu} - \eta_{\mu\nu}\xi \tag{15}$$

and, considering the gauge transform (Lorenz condition) with the condition

$$\Box\epsilon_\nu = \partial^\mu\bar{h}_{\mu\nu} \tag{16}$$

for the parameter ϵ^μ:

$$\partial^\mu\bar{h}_{\mu\nu} = 0, \tag{17}$$

the field equations can be rewritten like

$$\Box\bar{h}_{\mu\nu} = 0 \tag{18}$$

$$\Box\xi = -E^2\xi; \tag{19}$$

4

solutions of eqs. (18) and (19) are plan waves:

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p}) \exp(ip^{\alpha} x_{\alpha}) + c.c. \tag{20}$$

$$\xi = a(\overrightarrow{p}) \exp(iq^{\alpha} x_{\alpha}) + c.c. \tag{21}$$

where

$$k^{\alpha} \equiv (\omega, \overrightarrow{p}) \qquad \omega = p \equiv |\overrightarrow{p}|$$
$$\tag{22}$$
$$q^{\alpha} \equiv (\omega_E, \overrightarrow{p}) \qquad \omega_E = \sqrt{E^2 + p^2}.$$

In eqs. (18) and (20) the equation and the solution for the tensorial waves exactly like in General Relativity have been obtained [17], while eqs. (19) and (21) are respectively the equation and the solution for the scalar mode.

Note: the dispersion law for the modes of the scalar field ξ is not linear. The velocity of every tensorial mode $\bar{h}_{\mu\nu}$ is the light speed c, but the dispersion law (the second of eq. (22)) for the modes of ξ is that of a massive field which can be discussed like a wave-packet [12, 13]. Also, the group-velocity of a wave-packet of ξ centered in $\overrightarrow{p}$ is

$$\overrightarrow{v_G} = \frac{\overrightarrow{p}}{\omega}, \tag{23}$$

which is exactly the velocity of a massive particle with mass-energy E and momentum $\overrightarrow{p}$.

From the second of eqs. (22) and eq. (23) it is simple to obtain:

$$v_G = \frac{\sqrt{\omega^2 - E^2}}{\omega}. \tag{24}$$

Then, wanting a constant speed of the wave-packet, one needs

$$E = \sqrt{(1 - v_G^2)}\omega. \tag{25}$$

Now let us remain in the Lorenz gauge with trasformations of the type $\Box \epsilon_{\nu} = 0$; this gauge gives a condition of transversality for the tensorial part of the field: $k^{\mu} A_{\mu\nu} = 0$, but we do not know if the total field $h_{\mu\nu}$ is transverse. From eq. (15) it is

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\eta_{\mu\nu} - \eta_{\mu\nu}\xi. \tag{26}$$

At this point, if being in the massless case one could put

$$\Box \epsilon^{\mu} = 0$$
$$\tag{27}$$
$$\partial_{\mu} \epsilon^{\mu} = -\frac{\bar{h}}{2} - \xi,$$

5

which gives the total transversality of the field. But in the massive case this is impossible. In fact, if applying the d' Alembertian operator to the second of eqs. (27) and using the field equations (18) and (19) it is

$$\Box \epsilon^{\mu} = -E^2 \xi, \tag{28}$$

which is in contrast with the first of eqs. (27). In the same way it is possible to show that it does not exist any linear relation between the tensorial field $\bar{h}_{\mu\nu}$ and the scalar field ξ. Thus, a gauge in wich $h_{\mu\nu}$ is purely spatial cannot be chosen (i.e. we cannot put $h_{\mu 0} = 0$, see eq. (26)). But the traceless condition to the field $\bar{h}_{\mu\nu}$ can be put:

$$\Box \epsilon^{\mu} = 0$$
$$\partial_{\mu} \epsilon^{\mu} = -\frac{\bar{h}}{2}, \tag{29}$$

which implies

$$\partial^{\mu} \bar{h}_{\mu\nu} = 0. \tag{30}$$

Wanting to save the conditions $\partial_{\mu} \bar{h}^{\mu\nu}$ and $\bar{h} = 0$, transformations like

$$\Box \epsilon^{\mu} = 0$$
$$\partial_{\mu} \epsilon^{\mu} = 0 \tag{31}$$

can be used and, taking $\overrightarrow{p}$ in the z direction, one can choose a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different to zero. The condition $\bar{h} = 0$ gives $A_{11} = -A_{22}$. Now, putting these equations in eq. (26) and defining $\Phi \equiv -\xi$ one obtains

$$h_{\mu\nu}(t, z) = A^+(t - z)e^{(+)}_{\mu\nu} + A^{\times}(t - z)e^{(\times)}_{\mu\nu} + \Phi(t - v_G z)\eta_{\mu\nu}. \tag{32}$$

The term $A^+(t-z)e^{(+)}_{\mu\nu} + A^{\times}(t-z)e^{(\times)}_{\mu\nu}$ describes the two standard (i.e. tensorial) polarizations of gravitational waves which arises from General Relativity, while the term $\Phi(t - v_G z)\eta_{\mu\nu}$ is the scalar field.

3 An oscillating Cosmology

Before starting the analysis, one has to emphasize that, in a cosmologic framework, the linerar approximation has to be considered a good approximation. This arises from two considerations.

1. It is well known that, when treating cosmologic problems, astrophysicsts often limit their analyses to Newtonian theory. The linearized approximation represents the substitution of the Newtonian approximation with a less restrictive hypothesis. In this case, even if the field remains weak, it can vary with time and restrictions in the motion of test particles are not

present. New physics phenomena, in respect to Newtonian approximation, like light deflection and gravitational radiation, arises from this hypotesis. It is enlighting that, in the framework of standard General Relativity, Einstein used the linearized approach in the discussion about observative predictions of the theory [17].

2. As our observations are performed on Earth, the coordinate system in which the space-time is locally flat has to be used and the distance between any two points is given simply by the difference in their coordinates in the sense of post-Newtonian physics. This is exactly the sense of a spacetime which is considere *globally* curved but *locally* flat [17].

By assuming that, at cosmological scales, the third mode becomes dominant (i.e. $A^+, A^- \ll \Phi$) [7], as it appears from observations, eq. (32) can be rewritten as

$$h_{\mu\nu}(t, z) = \Phi(t, z)\eta_{\mu\nu} \tag{33}$$

and the corrispondent line element is the conformally flat one

$$ds^2 = [1 + \Phi(t, z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{34}$$

Defining

$$a^2 \equiv 1 + \Phi(t, z), \tag{35}$$

equation (34) becomes similiar to the well known cosmological Friedmann-Robertson Walker (FRW) line element of the standard homogeneus and isotropic flat Universe which is well known in the literature [16, 17, 18, 19, 20]:

$$ds^2 = [a^2(t, z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{36}$$

In the linearized approach it is also [7]

$$a \simeq 1 + \frac{1}{2}\Phi(t, z), \tag{37}$$

which shows that in the model the scale factor of the Universe oscillates near the (normalized) unity.

Below it will be shown that the model realizes an oscillating homogeneus and isotropic Universe, but, before starting with the analysis, we have to recall that observations today agrees with homogeneity and isotropy.

In Cosmology, the Universe is seen like a dynamic and thermodynamic system in which test masses (i.e. the "particles") are the galaxies that are stellar systems with a number of the order of $10^9 - 10^{11}$ stars [16, 17, 18, 19, 20]. Galaxies are located in clusters and super clusters, and observations show that, on cosmological scales, their distribution is uniform. This is also confirmed by the WMAP data on the Cosmic Background Radiation [21, 22]. These assumption can be summarized in the so called Cosmological Principle: *the Universe is homogeneous everyway and isotropic around every point.* Cosmological Principle

semplifies the analysis of the large scale structure, because it implies that the proper distances between any two galaxies is given by an universal scale factor which is the same for any couple of galaxies [16, 17, 18, 19, 20].

As our observations are performed in a laboratory environment on Earth, the coordinate system in which the space-time is locally flat has to be used and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics [12, 13, 14, 15, 16, 17]. This frame is the proper reference frame of a local observer, which we assume to be located on Earth. In this frame gravitational signals manifest themself by exerting tidal forces on the test masses, which are the galaxies of the Universe. A detailed analysis of the frame of the local observer is given in ref. [17], sect. 13.6. Here only the more important features of this coordinate system are recalled:

the time coordinate x_0 is the proper time of the observer O;

spatial axes are centered in O;

in the special case of zero acceleration and zero rotation the spatial coordinates x_j are the proper distances along the axes and the frame of the local observer reduces to a local Lorentz frame: in this case the line element reads [17]

$$ds^2 = -(dx^0)^2 + \delta_{ij}dx^i dx^j + O(|x^j|^2)dx^\alpha dx^\beta. \tag{38}$$

The effect of the gravitational force on test masses is described by the equation

$$\ddot{x}^i = -\widetilde{R}^i_{0k0}x^k, \tag{39}$$

which is the equation for geodesic deviation in this frame.

Thus, to study the effect of the third mode of the linearized Scalar-Tensor gravity on the galaxies, $\widetilde{R}^i_{0k0}$ has to be computed in the proper reference frame of the Earth. But, because the linearized Riemann tensor $\widetilde{R}_{\mu\nu\rho\sigma}$ is invariant under gauge transformations [12, 13, 17], it can be directly computed from eq. (33).

From [17] it is:

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\{\partial_\mu\partial_\beta h_{\alpha\nu} + \partial_\nu\partial_\alpha h_{\mu\beta} - \partial_\alpha\partial_\beta h_{\mu\nu} - \partial_\mu\partial_\nu h_{\alpha\beta}\}, \tag{40}$$

that, in the case eq. (33), begins

$$\widetilde{R}^\alpha_{0\gamma 0} = \frac{1}{2}\{\partial^\alpha\partial_0\Phi\eta_{0\gamma} + \partial_0\partial_\gamma\Phi\delta^\alpha_0 - \partial^\alpha\partial_\gamma\Phi\eta_{00} - \partial_0\partial_0\Phi\delta^\alpha_\gamma\}; \tag{41}$$

the different elements are (only the non zero ones will be written):

$$\partial^\alpha\partial_0\Phi\eta_{0\gamma} = \left\{ \begin{array}{ll} \partial_t^2\Phi & for \quad \alpha = \gamma = 0 \\ \\ -\partial_z\partial_t\Phi & for \quad \alpha = 3; \gamma = 0 \end{array} \right\} \tag{42}$$

$$\partial_0\partial_\gamma\Phi\delta_0^\alpha = \begin{cases} \partial_t^2\Phi & for \quad \alpha = \gamma = 0 \\ \\ \partial_t\partial_z\Phi & for \quad \alpha = 0; \gamma = 3 \end{cases} \tag{43}$$

$$-\partial^\alpha\partial_\gamma\Phi\eta_{00} = \partial^\alpha\partial_\gamma\Phi = \begin{cases} -\partial_t^2\Phi & for \quad \alpha = \gamma = 0 \\ \\ \partial_z^2\Phi & for \quad \alpha = \gamma = 3 \\ \\ -\partial_t\partial_z\Phi & for \quad \alpha = 0; \gamma = 3 \\ \\ \partial_z\partial_t\Phi & for \quad \alpha = 3; \gamma = 0 \end{cases} \tag{44}$$

$$-\partial_0\partial_0\Phi\delta_\gamma^\alpha = -\partial_t^2\Phi \quad for \quad \alpha = \gamma . \tag{45}$$

Now, putting these results in eq. (41), it results:

$$\widetilde{R}_{010}^1 = -\tfrac{1}{2}\ddot{\Phi}$$

$$\widetilde{R}_{020}^2 = -\tfrac{1}{2}\ddot{\Phi} \tag{46}$$

$$\widetilde{R}_{030}^3 = \tfrac{1}{2}(\partial_z^2\Phi - \partial_t^2\Phi).$$

But, the assumption of homogenity and isotropy implies $\partial_z\Phi = 0$ [7], which also implies

$$\widetilde{R}_{010}^1 = -\tfrac{1}{2}\ddot{\Phi}$$

$$\widetilde{R}_{0120}^2 = -\tfrac{1}{2}\ddot{\Phi} \tag{47}$$

$$\widetilde{R}_{030}^3 = -\tfrac{1}{2}\ddot{\Phi}$$

which show that the oscillations of the Universe are the same in any direction. Infact, using eq. (39), it results

$$\ddot{x} = \frac{1}{2}\ddot{\Phi}x, \tag{48}$$

$$\ddot{y} = \frac{1}{2}\ddot{\Phi}y \tag{49}$$

and

$$\ddot{z} = \frac{1}{2}\ddot{\Phi}z, \tag{50}$$

which are three perfectly symmetric oscillations.

9

4   Consistence with observations: i) The Hubble Law

The expansion of the Universe arises from the observations of E Hubble in 1929 [16, 17, 18, 19, 20]. The Hubble law states that, galaxies which are at a distance D, drift away from Earth with a velocity

$$v = H_0 D. \tag{51}$$

The today's Hubble expansion rate is

$$H_0 = h_{100} \frac{100 Km}{sec \times Mpc} = 3.2 \times 10^{-18} \frac{h_{100}}{sec}. \tag{52}$$

A dimensionless factor h_{100} is included, which now is just a convenience (in the past it came from an uncertainty in the value of H_0). From the WMAP data it is $h_{100} = 0.72 \pm 0.05$ [21, 22].

Calling f the frequency of the "cosmologic" gravitational wave and assuming that $f \ll H_0$ (i.e. the gravitational wave is "frozen" with respect the cosmological observations), the observations of Hubble and the more recent ones imply that our model of oscillating Universe has to be in the expansion phase [7]

A good way to analyse proper distances (which are equal to proper times in natural units) between two test masses is by means of light rays [12, 13, 17]. For the assumption of homogeneity and isotropy, only the radial propagation of the light can be taken into account.

In spherical coordinates equations (48), (49) and (50) give for the radial coordinate

$$\ddot{r} = \frac{1}{2} \ddot{\Phi} r. \tag{53}$$

Equivalently we can say that there is a gravitational potential

$$V(\overrightarrow{r}, t) = -\frac{1}{4} \ddot{\Phi}(t) r^2, \tag{54}$$

which generates the tidal forces, and that the motion of the test masses is governed by the Newtonian equation [7]

$$\ddot{\overrightarrow{r}} = - \bigtriangledown V. \tag{55}$$

Because we are in the linearized theory, following [7], the solution of eq. (53) can be found by using the perturbation method [16, 17], obtaining

$$D = D_0 + \frac{1}{2} D_0 \Phi(t) = (1 + \frac{1}{2}\Phi) D_0 = a(t) D_0 \tag{56}$$

Deriving this equation with respect the time we also get

$$\frac{dD}{dt} = D_0 \frac{da(t)}{dt}. \tag{57}$$

Thus the Hubble law is obtained:

$$\frac{1}{D}\frac{dD}{dt} = H_0, \qquad (58)$$

where

$$H_0 = (\frac{1}{a}\frac{da}{dt})_0. \qquad (59)$$

5 Consistence with observations: ii) The Cosmological Redshift

Let us now consider another point of view. The conformal line element (36) can be put in spherical coordinates, obtaining [7]

$$ds^2 = [1 + \Phi(t)](-dt^2 + dr^2). \qquad (60)$$

In this line element the angular coordinates have been neglected because of the assumption of homogeneity and isotropy. The condition of null geodesic in the above equation gives

$$dt^2 = dr^2. \qquad (61)$$

Thus, from eq. (61), it results that the coordinate velocity of the photon in the gauge (60) is equal to the speed of light. This because in the coordinates (60) t is only a time coordinate. The rate $d\tau$ of the proper time (distance) is related to the rate dt of the time coordinate from [16]

$$d\tau^2 = g_{00}dt^2. \qquad (62)$$

From eq. (60) it is $g_{00} = (1 + \Phi)$. Then, using eq. (61), we obtain

$$d\tau^2 = (1 + \Phi))dr^2, \qquad (63)$$

which gives

$$d\tau = \pm[(1 + \Phi)]^{\frac{1}{2}}dr \simeq \pm[(1 + \frac{1}{2}\Phi)]dr. \qquad (64)$$

We assume that photons are travelling by the galaxy to Earth in this case too, thus the negative sign is needed [7].

Integrating this equation it is

$$\int_{\tau_1}^{\tau_0} \frac{d\tau}{1 + \frac{1}{2}\Phi(t)} = \int_{r_g}^{0} dr = r_g, \qquad (65)$$

where τ_1 and τ_0 are the emission and reception istants of the photon from galaxy and Earth respectively. If the light is emitted with a delay $\triangle\tau_1$, it arrives on Earth with a delay $\triangle\tau_0$. In this way

$$\int_{\tau_1}^{\tau_0} \frac{d\tau}{1 + \frac{1}{2}\Phi(t)} = \int_{\tau_1+\triangle\tau_1}^{\tau_0+\triangle\tau_0} \frac{d\tau}{1 + \frac{1}{2}\Phi(t)} = r_g. \qquad (66)$$

11

The radial coordinate r_g is *comoving* (i.e. constant in the gauge (60)) because the assumption of homogenity and isotropy implies $\partial_z \Phi = 0$, which removes the z dependence in the line element (36). Thus the only dependence in the line element (60) is the t dependence in the scale factor $a = 1 + \frac{1}{2}\Phi(t)$. Then, from equation (66) it is

$$\int_{\tau_1}^{\tau_0} \frac{d\tau}{1+\frac{1}{2}\Phi(t)} = \int_{\tau_1}^{\tau_0} \frac{d\tau}{1+\frac{1}{2}\Phi(t)} +$$
$$+ \int_{\tau_1}^{\tau_0+\triangle\tau_0} \frac{d\tau}{1+\frac{1}{2}\Phi(t)} - \int_{\tau_0}^{\tau_1+\triangle\tau_1} \frac{d\tau}{1+\frac{1}{2}\Phi(t)}, \tag{67}$$

which gives

$$\int_{\tau_1}^{\tau_0+\triangle\tau_0} \frac{d\tau}{1+\frac{1}{2}\Phi(t)} = \int_{\tau_0}^{\tau_1+\triangle\tau_1} \frac{d\tau}{1+\frac{1}{2}\Phi(t)}. \tag{68}$$

This equation can be semplified, obtaining [7]

$$\int_0^{\triangle\tau_0} \frac{d\tau}{1+\frac{1}{2}\Phi(t)} = \int_0^{\triangle\tau_1} \frac{d\tau}{1+\frac{1}{2}\Phi(t)}, \tag{69}$$

which gives

$$\frac{\triangle\tau_0}{1+\frac{1}{2}\Phi(t_0)} = \frac{\triangle\tau_1}{1+\frac{1}{2}\Phi(t_1)}. \tag{70}$$

Then

$$\frac{\triangle\tau_1}{\triangle\tau_0} = \frac{1+\frac{1}{2}\Phi(t_1)}{1+\frac{1}{2}\Phi(t_0)} = \frac{a(t_1)}{a(t_0)}. \tag{71}$$

But frequencies are inversely proportional to times, thus

$$\frac{f_0}{f_1} = \frac{\triangle\tau_1}{\triangle\tau_0} = \frac{1+\frac{1}{2}\Phi(t_1)}{1+\frac{1}{2}\Phi(t_0)} = \frac{a(t_1)}{a(t_0)}. \tag{72}$$

If one recalls the definition of the *redshift parameter* [16, 17, 18, 19, 20]

$$z \equiv \frac{f_1 - f_0}{f_0} = \frac{\triangle\tau_0 - \triangle\tau_1}{\triangle\tau_1}, \tag{73}$$

using equation (71), equation (73) gives

$$z = \frac{a(t_0)}{a(t_1)} - 1, \tag{74}$$

which is well known in the literature [16, 17, 18, 19, 20].

Thus, it has been shown that the model is fine-tuned with the Hubble Law and the Cosmological Redshift, exactly like the previous model in [7].

6 Conclusions

An oscillating, homogeneous and isotropic Universe which arises by the linearized Scalar-Tensor gravity has been discussed, integrating my previous research in [7] and showing that some observative evidences, like the Cosmological Redshift and the Hubble law, are fine-tuned with the model in this case too. In this context Dark Energy appears like a pure curvature effect arising by the scalar field.

7 Acknowledgements

References

[1] Peebles PJE and Ratra B - Rev. Mod. Phys. **75** 8559 (2003)

[2] Starobinsky AA - Phys. Lett. B **91**, 99 (1980)

[3] Allemandi G, Francaviglia M, Ruggiero ML and Tartaglia A - Gen. Rev. Grav. **37** 11 (2005);Capozziello S and Troisi A - Phys. Rev. D **72** 044022 (2005)

[4] Inagaky T, Nojiri S and Odintsov SD - J. Cosmol. Astropart. Phys. JCAP0506010 (2005); Allemandi G, Capone M, Capozziello S and Francaviglia M - Gen. Rev. Grav. **38** 1 (2006)

[5] Capozziello S, Cardone VF and Troisi A - J. Cosmol. Astropart. Phys. JCAP08001 (2006); Nojiri S, Odintsov SD and Tretyakov P - arXiv: 07105232 (2007)

[6] Capozziello S - *Newtonian Limit of Extended Theories of Gravity* in *Quantum Gravity Research Trends* Ed. A. Reimer, pp. 227-276 Nova Science Publishers Inc., NY (2005) - also in arXiv:gr-qc/0412088 (2004) ; Nojiri S and Odintsov SD - hep-th/0601213 (2006)

[7] Corda C - Gen. Rel. Grav. DOI: 10.1007/s/10714-088-0627-3; also in arXiv: 0802.2523 (2008);

[8] Will C M *Theory and Experiments in Gravitational Physics*, Cambridge Univ. Press Cambridge (1993); Corda C - Mod. Phys. Lett. A 23, 2, 109-114 (2008)

[9] Capozziello S, Cardone VF and Troisi A - Phys. Rev. D **71** 043503 (2005)

[10] Corda C - Astropart. Phys. **27,** No 6, 539-549 (2007); Hild S (for the LIGO Scientific Collaboration) - Class. Quant. Grav. **23** 19 S643-S651 (2006)

[11] Acernese F et al. (the Virgo Collaboration) - Class. Quant. Grav. **23** 19 S635-S642 (2006); Tatsumi D, Tsunesada Y and the TAMA Collaboration - Class. Quant. Grav. **21** 5 S451-S456 (2004)

[12] Capozziello S and Corda C - Int. J. Mod. Phys. D **15** 1119 - 1150 (2006); Corda C - *Response of laser interferometers to scalar gravitational waves*- talk in the *Gravitational Waves Data Analysis Workshop in the General Relativity Trimester of the Institut Henri Poincare* - Paris 13-17 November 2006, on the web in www.luth2.obspm.fr/IHP06/workshops/gwdata/corda.pdf

[13] Corda C - J. Cosmol. Astropart. Phys. JCAP04009 (2007)

[14] Tobar ME, Suzuki T and Kuroda K Phys. Rev. D 59 102002 (1999);

[15] Corda C - Mod. Phys. Lett. A **22** No. 23 (2007) 1727-1735;

[16] Landau L and Lifsits E - "Teoria dei campi" - Editori Riuniti edition III (1999)

[17] Misner CW, Thorne KS and Wheeler JA - "Gravitation" - W.H.Feeman and Company - 1973

[18] Peebles PJE - "Large Scale Structure of the Universe" - Princeton University Press, Princeton (1980)

[19] Peebles PJE - "Principles of Physical Cosmology" - Princeton University Press, Princeton (1993)

[20] Capozziello S and Funaro M - "Introduzione alla Relatività Generale " - Liguori Editori, Napoli (2005)

[21] C. L. Bennet and others - ApJS **148** 1 (2003)

[22] D. N. Spergel and others - ApJS **148** 195 (2003)

[23] Brans C and Dicke RH - Phys. Rev. 124, 925 (1961)